A Graphical User Interface (GUI) for Plane Wave Scattering from a Conducting, Dielectric or a Chiral Sphere

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Abstract
Various numerical techniques have been developed for modeling the electromagnetic field propagation in various novel complex media. The validity of these techniques is usually verified by comparison to the exact solutions of canonical problems. Recently, chiral medium, which is a subclass of materials known as bianisotropic materials, has gained focus in research and numerical techniques have been developed in order to calculate interaction of fields with chiral objects. One canonical problem for these techniques is the plane wave scattering from a chiral sphere. This paper presents a software package that displays and saves the calculated data for the scattering from a chiral, dielectric or a perfectly conducting sphere using a friendly graphical user interface (GUI).

Introduction

The interaction of electromagnetic fields with chiral matters has been studied over the years. Chiral media were used in many applications involving antennas and arrays, antenna radomes, microstrip substrates and waveguides. A chiral object is, by definition, a body that lacks bilateral symmetry, which means that it cannot be superimposed on its mirror image neither by translation nor rotation. This is also known as handedness. Objects that have the property of handedness are said to be either right-handed or left-handed. Chiral media are optically active – a property caused by asymmetrical molecular structure that enables a substance to rotate the plane of incident polarized light, where the amount of rotation in the plane of polarization is proportional to the propagation distance through the medium as well as to the light wavelength [1-5]. Chiral medium is therefore has an effect on the attenuation rate of the right hand and left hand circularly polarized waves. Unlike dielectric or conducting cylinders, chiral scatterers produce both co-polarized and cross-polarized scattered fields. Coating with chiral material is therefore attempted for reducing radar cross-section of targets.

The electromagnetic wave propagation in chiral and bi-isotropic media has been modeled by various numerical techniques recently in various studies. In most of these studies, the validity of the developed techniques is verified by comparing the numerical results to the results of one-dimensional and two-dimensional problems that has known exact solutions. For the techniques solving three-dimensional problems, plane wave scattering from a chiral sphere is the benchmark. The exact analytical solution of the scattering by a chiral sphere has been introduced by Bohren [6] and a detailed analysis of the solution is given by Worasawate [7]. This formulation has been used for verification of the scattering from arbitrary shaped 3D chiral objects using Method of Moments analysis [8] and Finite Difference Time Domain Analysis [9].

In this contribution, a software package is developed and presented to calculate plane wave scattering from a chiral sphere. The package involves a user-friendly GUI, which enables the user to enter the scattering parameters and observe the results, in near real time, and save the calculated data and displayed figures. As will be discussed in the following sections, due to the nature of the chiral constitutive relations, the developed program can be used to calculate scattering from a dielectric or a perfectly conducting sphere as well. The presented program is based on the exact solution provided in [7] which is summarized here for the reader's convenience.

Plane Wave Scattering from A Chiral Sphere

The constitutive relations for a chiral media can be written as

$$\bar{D} = \varepsilon\bar{E} - j\kappa\sqrt{\mu_0\varepsilon_0}\,\bar{H} \tag{1}$$

$$\bar{B} = \mu\bar{H} + j\kappa\sqrt{\mu_0\varepsilon_0}\,\bar{E} \tag{2}$$

where κ is the chirality parameter. Equations (1) and (2) can be written alternatively as

$$\bar{D} = \varepsilon\bar{E} - j\xi\bar{H} \tag{3}$$

$$\bar{B} = \mu\bar{H} + j\xi\bar{E} \tag{4}$$

where ξ_r is the relative chirality. The relative chirality is defined as $\xi_r = \dfrac{\xi}{\sqrt{\mu\varepsilon}} = \dfrac{\kappa}{\sqrt{\mu_r\varepsilon_r}}$.

The electromagnetic field in a chiral medium can be decomposed into two parts, the right-handed wave ($\bar{E}_+$, $\bar{H}_+$) and the left-handed wave ($\bar{E}_-$, $\bar{H}_-$). These waves see the chiral medium as equivalent isotropic media characterized by $(\varepsilon_\pm, \mu_\pm)$. Electric displacement vectors $\bar{D}_\pm$, magnetic flux densities $\bar{B}_\pm$, and wave impedances $\eta_\pm$ for the equivalent media are defined by

$$\bar{D}_\pm = \varepsilon_\pm\bar{E}_\pm \tag{5}$$

$$\bar{B}_\pm = \mu_\pm\bar{H}_\pm \tag{6}$$

$$\eta_\pm = \sqrt{\frac{\mu_\pm}{\varepsilon_\pm}} = \sqrt{\frac{\mu}{\varepsilon}} = \eta = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} \tag{7}$$

where $\mu = \mu_o\mu_r$, $\varepsilon = \varepsilon_o\varepsilon_r$, and $\eta_o = \sqrt{\dfrac{\mu_o}{\varepsilon_o}}$ is the free space wave impedance, and

$$\varepsilon_{\pm} = \varepsilon \pm \frac{\xi}{\eta} \tag{8}$$

$$\mu_{\pm} = \mu_{\pm} \pm \xi\eta \,. \tag{9}$$

The electromagnetic fields ($\overline{E}, \overline{H}$) are the sum of the right-handed waves ($\overline{E}_{+}, \overline{H}_{+}$) and the left-handed waves ($\overline{E}_{-}, \overline{H}_{-}$) as

$$\overline{E} = \overline{E}_{+} + \overline{E}_{-} \tag{10}$$

$$\overline{H} = \overline{H}_{+} + \overline{H}_{-} \tag{11}$$

where

$$\overline{E}_{\pm} = \frac{1}{2}[\overline{E} \mp j\eta\overline{H}] = \mp j\eta H_{\pm} \tag{12}$$

$$\overline{H}_{\pm} = \frac{1}{2}[\overline{H} \pm \frac{j}{\eta}\overline{E}] = \pm \frac{j\overline{E}_{\pm}}{\eta} \,. \tag{13}$$

Maxwell equations in a source free region for the equivalent media are

$$\nabla \times \overline{E}_{\pm} = \pm k_{\pm}\overline{E}_{\pm} = -j\omega\mu_{\pm}\overline{H}_{\pm} \tag{14}$$

$$\nabla \times \overline{H}_{\pm} = \pm k_{\pm}\overline{H}_{\pm} = +j\omega\varepsilon_{\pm}\overline{E}_{\pm} \tag{15}$$

where $k_{\pm}$ are the wave numbers for the chiral media, given in terms of the free space wavenumber $k_o = \omega\sqrt{\mu_o\varepsilon_o}$ as

$$k_{\pm} = \omega\sqrt{\mu_{\pm}\varepsilon_{\pm}} = k_o\sqrt{\mu_r\varepsilon_r}(1 \pm \xi_r). \tag{16}$$

The spherical vector wave functions $\overline{M}^{(i)}_{\{e,o\}mn}$ and $\overline{N}^{(i)}_{\{e,o\}mn}$ required for the representation of the fields in spherical coordinates are

$$\begin{aligned}\overline{M}^{(i)}_{\{e,o\}mn}(k_{\pm}r) = \hat{a}_{\theta}\frac{\hat{B}_n(k_{\pm}r)}{k_{\pm}r}\frac{mP_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi),\cos(m\phi)\} \\ +\hat{a}_{\phi}\frac{\hat{B}_n(k_{\pm}r)}{k_{\pm}r}\sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi),\sin(m\phi)\}\end{aligned} \tag{17}$$

$$\bar{N}^{(i)}_{\{e,o\}mn}(k_\pm r) = \hat{a}_r n(n+1)\frac{\hat{B}_n(k_\pm r)}{(k_\pm r)^2} P_n^m(\cos\theta)\{\cos(m\phi),\sin(m\phi)\}$$
$$-\hat{a}_\theta \frac{\hat{B}'_n(k_\pm r)}{k_\pm r}\sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi),\sin(m\phi)\} \tag{18}$$
$$+\hat{a}_\phi \frac{\hat{B}'_n(k_\pm r)}{k_\pm r}\frac{mP_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi),\cos(m\phi)\}$$

where

$$\hat{B}_n(z) = zb_n(z)\,, \tag{19}$$

$$\hat{B}'_n(z) = \frac{d}{dz}(zb_n(z))\,, \tag{20}$$

$$P_n^{m\prime}(x) = \frac{d}{dx}(P_n^m(x))\,, \tag{21}$$

while P_n^m is the associated Legendre polynomial of order m and degree n and the superscript *(i)* indicates the choice of the spherical Bessel function $b_n(kr)$. As $b_n(kr)$ is $j_n(kr)$ when $i=1$, $b_n(kr)$ is $y_n(kr)$ when $i=2$, $b_n(kr)$ is $h_n^{(1)}(kr)$ when $i=3$ and $b_n(kr)$ is $h_n^{(2)}(kr)$ when $i=4$. Because the field components should be finite at the origin, only the terms for which $i=1$ are used in the solutions for the fields inside the sphere, and for the scattered field in the region outside the sphere, only terms for which $i=4$ are used in the solutions to satisfy the radiation conditions. The incident plane wave can be represented in terms of the spherical vector wave functions in order to apply the appropriate boundary conditions. Therefore, considering an x-polarized and z-traveling incident plane wave such that

$$\bar{E}^{inc} = \hat{a}_x E_o e^{-jk_o z} = \hat{a}_x E_o e^{-jk_o r\cos\theta} \tag{22}$$

$$\bar{H}^{inc} = \hat{a}_y \frac{E_o}{\eta_o} e^{-jk_o z} = \hat{a}_y \frac{E_o}{\eta_o} e^{-jk_o r\cos\theta} \tag{23}$$

and after some mathematical manipulations the incident electric and magnetic field vectors can be written in terms of spherical vector wave functions as

$$\bar{E}^{inc} = -E_o \sum_{n=1}^{\infty}\left\{\frac{j^{-n}(2n+1)}{n(n+1)}\left(\bar{M}^{(1)}_{o1n}(k_o r) + j\bar{N}^{(1)}_{e1n}(k_o r)\right)\right\} \tag{24}$$

$$\bar{H}^{inc} = \frac{E_o}{\eta_o} \sum_{n=1}^{\infty}\left\{\frac{j^{-n}(2n+1)}{n(n+1)}\left(\bar{M}^{(1)}_{e1n}(k_o r) - j\bar{N}^{(1)}_{o1n}(k_o r)\right)\right\}. \tag{25}$$

Upon using equations (17) and (18) with i = 4, the scattered field vectors $\bar{E}^s$ and $\bar{H}^s$ are given by

$$\bar{E}^s = -E_o \sum_{m,n} \frac{j^{-n}(2n+1)}{n(n+1)} \left\{ \left(ja_{mn} \bar{N}^{(4)}_{emn}(k_o r) + jb_{mn} \bar{N}^{(4)}_{omn}(k_o r) + c_{mn} \bar{M}^{(4)}_{emn}(k_o r) + d_{mn} \bar{M}^{(4)}_{omn}(k_o r) \right) \right\} \tag{26}$$

$$\bar{H}^s = \frac{E_o}{\eta_o} \sum_{m,n} \frac{j^{-n}(2n+1)}{n(n+1)} \left\{ \left(-jc_{mn} \bar{N}^{(4)}_{emn}(k_o r) - jd_{mn} \bar{N}^{(4)}_{omn}(k_o r) + a_{mn} \bar{M}^{(4)}_{emn}(k_o r) + b_{mn} \bar{M}^{(4)}_{omn}(k_o r) \right) \right\} \tag{27}$$

while upon using equations (17) and (18) with i = 1, the fields inside the chiral sphere $\bar{E}^{chiral}$ and $\bar{H}^{chiral}$ are given by

$$\begin{aligned} \bar{E}^{chiral} = -E_o \sum_{m,n} \frac{j^{-n}(2n+1)}{n(n+1)} \Big\{ & jg_{mn} \left(\bar{N}^{(1)}_{emn}(k_+ r) + \bar{M}^{(1)}_{emn}(k_+ r) \right) \\ & + jh_{mn} \left(\bar{N}^{(1)}_{omn}(k_+ r) + \bar{M}^{(1)}_{omn}(k_+ r) \right) \\ & + u_{mn} \left(\bar{N}^{(1)}_{emn}(k_- r) - \bar{M}^{(1)}_{emn}(k_- r) \right) + w_{mn} \left(\bar{N}^{(1)}_{omn}(k_- r) - \bar{M}^{(1)}_{omn}(k_- r) \right) \Big\} \end{aligned} \tag{28}$$

$$\begin{aligned} \bar{H}^{chiral} = \frac{E_o}{\eta_o} \sum_{m,n} \frac{j^{-n}(2n+1)}{n(n+1)} \Big\{ & g_{mn} \left(\bar{N}^{(1)}_{emn}(k_+ r) + \bar{M}^{(1)}_{emn}(k_+ r) \right) \\ & + h_{mn} \left(\bar{N}^{(1)}_{omn}(k_+ r) + \bar{M}^{(1)}_{omn}(k_+ r) \right) \\ & + ju_{mn} \left(\bar{N}^{(1)}_{emn}(k_- r) - \bar{M}^{(1)}_{emn}(k_- r) \right) + jw_{mn} \left(\bar{N}^{(1)}_{omn}(k_- r) - \bar{M}^{(1)}_{omn}(k_- r) \right) \Big\}. \end{aligned} \tag{29}$$

The scattered electromagnetic field in the presence of a chiral sphere of radius $r = a$ can be obtained using (24)-(29). These equations are used to construct a set of simultaneous equations to solve for the unknown coefficients a_{mn}, b_{mn}, c_{mn}, d_{mn}, g_{mn}, h_{mn}, u_{mn} and w_{mn}. The incident field excitations contain only the terms for which m = 1. Therefore, only the m = 1 terms are included in the solutions for the scattered field and the electromagnetic field inside the chiral sphere. Thus by applying the boundary conditions that require the tangential components of the electric and magnetic fields be continuous at $r = a$ and after some manipulations the unknowns a_{1n}, b_{1n}, c_{1n}, d_{1n} are found as

$$a_{1n} = \frac{(ERB - FA)(CH - RGD) + (CER - GA)(HB - RFD)}{\Delta_1} \tag{30}$$

$$b_{1n} = \frac{R(CF - BG)}{(k_o a)^2 \Delta_1} \tag{31}$$

$$c_{1n} = -b_{1n} \tag{32}$$

$$d_{1n} = \frac{(ARG - CE)(FD - RBH) + (ARF - BE)(GD - RCH)}{\Delta_1} \tag{33}$$

where

$$\Delta_1 = (CH - RGD)(FD - RBH) + (GD - RCH)(HB - RFD)$$

$$A = \frac{\hat{J}_n(k_o a)}{k_o a},\ B = \frac{\hat{J}_n(k_+ a)}{k_+ a},\ C = \frac{\hat{J}_n(k_- a)}{k_- a},\ D = \frac{\hat{H}_n^{(2)}(k_o a)}{k_o a}$$

$$E = \frac{\hat{J}'_n(k_o a)}{k_o a},\ F = \frac{\hat{J}'_n(k_+ a)}{k_+ a},\ G = \frac{\hat{J}'_n(k_- a)}{k_- a}, H = \frac{\hat{H}_n^{(2)\prime}(k_o a)}{k_o a},\ R = \frac{\eta_o}{\eta}$$

while $\hat{B}_n(z) = \sqrt{\frac{\pi z}{2}} B_{n+\frac{1}{2}}$ and $B_{n+\frac{1}{2}}$ is a cylindrical Bessel function. The co-polarized bistatic radar cross section $\sigma_{\theta\theta}$ and the cross-polarized bistatic radar cross section $\sigma_{\phi\theta}$ can then be defined as

$$\sigma_{\theta\theta} = \lim_{r \to \infty} 4\pi r^2 \frac{\left|E_\theta^s\right|^2}{\left|E_\theta^{inc}\right|^2} \tag{34}$$

$$\sigma_{\phi\theta} = \lim_{r \to \infty} 4\pi r^2 \frac{\left|E_\phi^s\right|^2}{\left|E_\theta^{inc}\right|^2}\,. \tag{35}$$

With the assumption that the plane of interest is defined by $\phi = 0^o$ one can obtain

$$\sigma_{\theta\theta} = \frac{\lambda_o^2}{\pi} \left| \sum_{n=1}^{\infty} \left\{ \frac{2n+1}{n(n+1)} \left(a_{1n}\tau_n + d_{1n}\pi_n \right) \right\} \right|^2 \tag{36}$$

$$\sigma_{\phi\theta} = \frac{\lambda_o^2}{\pi} \left| \sum_{n=1}^{\infty} \left\{ \frac{2n+1}{n(n+1)} \left(b_{1n}\pi_n - c_{1n}\tau_n \right) \right\} \right|^2 \tag{37}$$

where

$$\pi_n = \frac{P_n^1(\cos\theta)}{\sin\theta} \tag{38}$$

$$\tau_n = -\sin\theta P_n^{1\prime}(\cos\theta)\,. \tag{39}$$

Software Description

A program is developed to calculate the scattered fields from a chiral sphere due to an incident x-polarized and z-traveling plane wave. If the chirality vanishes, that is $\kappa = 0$, the constitutive relations given in (1) and (2) reduce to that of a dielectric medium. Therefore, this program can be used to calculate the scattering from a dielectric sphere as well. Furthermore, if a very large value of the dielectric constant is used, the medium behaves like a highly conductive medium. Thus, this program also can calculate the scattering from a highly conductive or a PEC sphere.

A graphical user interface is developed using Matlab in order to provide a user-friendly environment for the calculation and visualization of the results. A snapshot of this user interface is shown in Fig. 1. The user can choose to calculate scattering from a chiral, dielectric or a conductive sphere from a drop-down menu. The radius of the sphere, relative permittivity, relative permeability, and chirality parameters can be entered through entry boxes. The unnecessary parameters are not going to be used during calculations; such that chirality will not be used if dielectric sphere is selected, and permittivity, permeability and chirality are not to be used if conductive sphere is selected. The frequency of the incident field is another parameter that should be supplied by the user. One of four types of results can be viewed in the plot window; co-polarized bistatic radar cross-section $\sigma_{\theta\theta}$, cross-polarized bistatic radar cross-section $\sigma_{\phi\theta}$, magnitude of E_θ, and magnitude of E_ϕ. The field components E_θ and E_ϕ are calculated at a specified distance from the center of the sphere that should be entered by the user. Therefore, the near as well as far field components E_θ and E_ϕ can be calculated and displayed. The radar cross-section values $\sigma_{\theta\theta}$ and $\sigma_{\phi\theta}$ are computed from far field components regardless of the distance value entered by the user. These fields are calculated on an arc defined by $0^o \le \theta \le 180^o$, $\phi = \phi_o$ in spherical coordinates. The angle ϕ_o is another parameter that should be entered by the user.

As can be seen in equation (26), the solution of the scattered fields is a sum of an infinite series. The user can enter the number of terms to be used for calculation, however the calculation will converge to exact results only with large enough number of terms. The program provides the user the option to display the solution convergence in terms of the number of terms by a radio button. This feature would allow the user to examine the convergence of such complicated summations.

The parameter entry boxes have sliding bars in order to provide a range for each input parameter to the program. Once the parameter is changed with the slider, the corresponding calculations are performed and displayed in the result window. Therefore, the variation of results with respect to parameters can be viewed in a near real time manner.

Any displayed data in the result window can be saved to a Matlab script file, together with the Matlab plotting commands. Once a plot is saved, it can be displayed again later by running the saved script file in Matlab. Numerical data can be extracted from this file if the user wishes to use it in another package.

This package is developed and tested using Matlab version 14, and a p-coded version of this package is available for free download from the Applied Computational Electromagnetic Society (ACES) web site (http://aces.ee.olemiss.edu).

Conclusions

An interactive software package is developed to calculate and display the scattered fields and radar cross-sections of a chiral, a dielectric or a perfectly conducting sphere due to an incident plane wave.

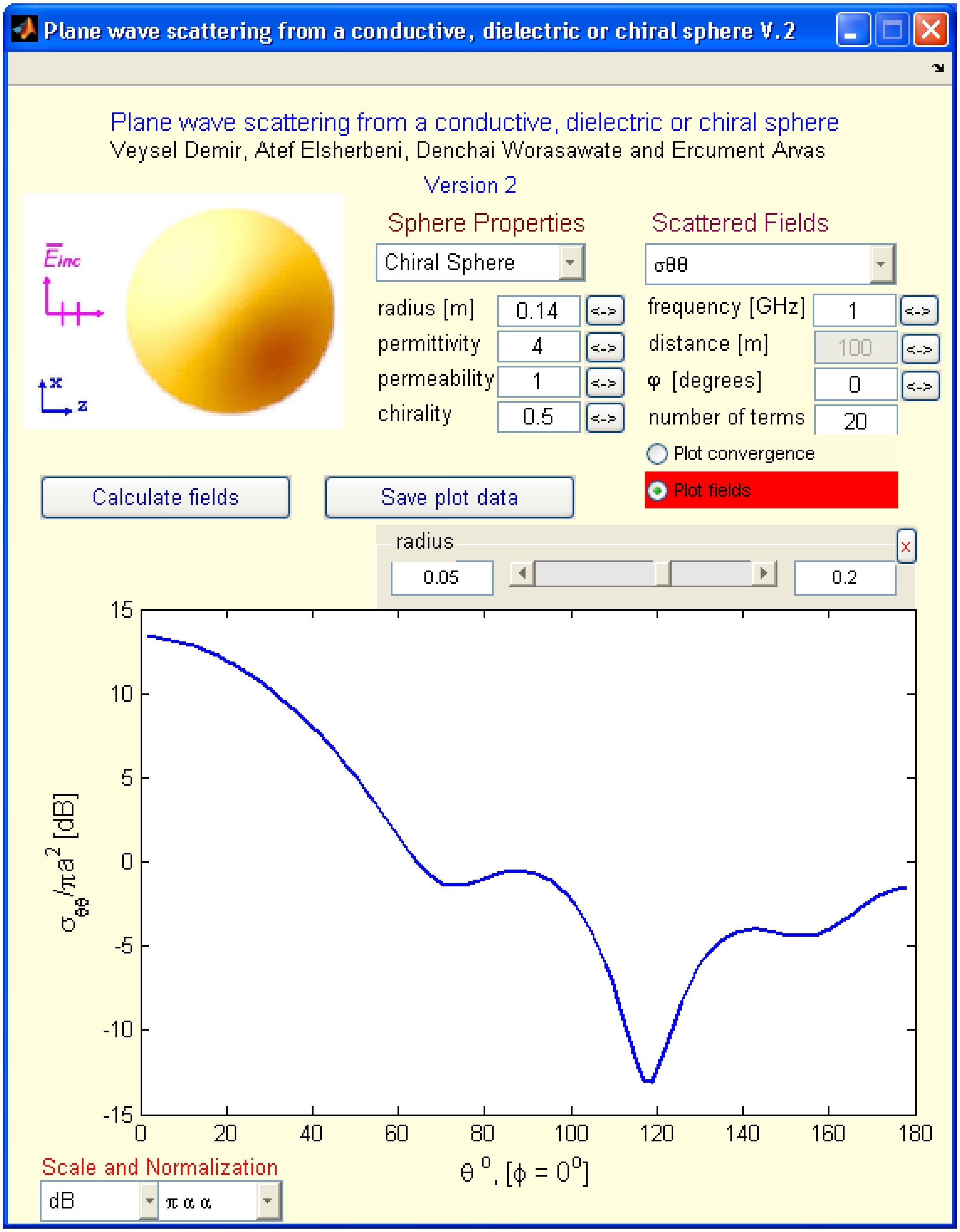

Figure 1. A GUI for plane wave scattering from a chiral, dielectric or a PEC sphere.

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