Sub-Gridding Errors in Standard and Hybrid Higher Order FDTD Simulations

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Introduction

- What is the **Finite-Difference Time-Domain Method (FDTD)**?
- What is it used for?
- Why do we need Subgridding?
**Problem:** Difficulties with 5G and IoT Device Design

**Proposed Solution:** Subgridding can be used to save memory and CPU time while maintaining an accurate solution.

https://www.ursalink.com/en/blog/5g-iot
Main Research Objectives

- The repercussions of subgridding in a FDTD calculation can lead to dispersion and stability errors [1-3].
- A larger subgrid enhances the maximum area an object of interest can be meshed to receive a more accurate analysis in a local grid. The deleterious effects of larger subgridding ratios have been discussed in the literature [4].
- A topic that has not yet been investigated, is the relative error that arises with increased electrical sizes of subgridded regions, independent of the contrast ratio.
- This research will focus on the effect the size of a subgridded region has on the resulting errors with 1:3, 1:9, 1:15, and 1:27 contrast ratios within 1D and 2D FDTD simulations.

Superimposed Coarse and Fine Grid – TMz Case

$(i_c, j_c) = $ coarse grid indices

$(i_f, j_f) = $ fine grid indices

$(l_f, l_f) = $ coarse grid relationship to beginning of fine grid

$E_{zc} = $ boundary

$H_{xc} = $ boundary

$H_{yc} = $ boundary

$e_{zf} = $ boundary

$h_{xf} = $ boundary

$h_{xf} = $ boundary

$h_{yf} = $ boundary

$I_z, J_z = $ coarse grid relationship to beginning of fine grid

$i_z, j_z = $ coarse grid indices

$i_z, j_z = $ fine grid indices
1. Update $H_{xc}$ everywhere in the coarse grid.
2. Update $h_{xf}$ everywhere in the fine grid using updating equation.
3. Update only boundary $H_{xc}$ with the new value for $h_{xf}$ at specific collocated locations.
4. Update $H_{yc}$ everywhere in the coarse grid.
5. Update $h_{yf}$ everywhere in the fine grid using updating equation.
6. Update only boundary $H_{yc}$ with the new value for $h_{yf}$ at specific collocated locations.
7. Update $E_{zc}$ everywhere in the coarse grid.
8. Update the collocated $e_{zf}$ with the value of $E_{zc}$.
9. Interpolation of non-collocated $e_{zf}$
10. Update non-boundary $e_{zf}$ using fine grid magnetic fields.
11. Repeat steps 1-10 for all following time steps.
Step 9 - $e_{zf}$, boundary interpolation method

9. Update $e_{zf}$ only along boundary using interpolation of $E_{zc}$.  
   a) Corner & Edge boundaries  
      i. Interpolate between two closest $E_{zc}$ coarse nodes.  
      ii. Fine grid nodes, $e_{zf}$, will receive 2/3 the value of the node closest (1 fine grid step away) and it will receive 2/3 of the next closest coarse node (2 fine grid steps away). Equation 1.

$$
\begin{align*}
e_{zf}^{n+1}(I_f + i_f, J_f + j_f) &= \frac{\text{fine grid steps to closest coarse node}}{3} E_{zc}^{n+1}(I_f + i_f, J_f + j_f) \\
+ \frac{\text{fine grid steps to next closest coarse node}}{3} E_{zc}^{n+1}(I_f + i_f, J_f + j_f) 
\end{align*}
$$

where,

$(I_f, J_f)$ = index for the beginning of the fine grid in terms of the coarse grid coordinates,
$(i_f, j_f)$ = indices of fine grid component locations within the fine grid

2D Subgridding – Single Subgrid Region
Normalized Difference 2D Calculation Equation

\[ Max \text{ Normalized Difference}(i, j, t) = \frac{|E_{zSubgrid}(i, j, t) - E_{zReference}(i, j, t)|_{Max}}{|E_{zReference}(i, j, t)|_{Max}} \times 100 \]

for
\[ t = \text{all time} \]
\[ i = [1: nx + 1] \]
\[ j = [1: ny + 1] \]
**Problem Space**

*2D Domain:* 308 x 243 Coarse Cells

**SUBGRID REGION:**
143 x 30 Coarse Cells

<table>
<thead>
<tr>
<th>Contrast Ratio</th>
<th>Coarse Cell Size $(dx = dy)$ (mm)</th>
<th>Fine Cell Size $(dx_{fine} = dy_{fine})$ (mm)</th>
<th>Time Step Size $(dt)$ (ps)</th>
<th>Number of Time Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:3</td>
<td>3 mm</td>
<td>1 mm</td>
<td>2.1 ps</td>
<td>3,000</td>
</tr>
<tr>
<td>1:9</td>
<td>3 mm</td>
<td>0.33 mm</td>
<td>0.7 ps</td>
<td>9,000</td>
</tr>
<tr>
<td>1:15</td>
<td>3 mm</td>
<td>0.2 mm</td>
<td>0.42 ps</td>
<td>15,000</td>
</tr>
<tr>
<td>1:27</td>
<td>3 mm</td>
<td>0.11 mm</td>
<td>0.24 ps</td>
<td>27,000</td>
</tr>
</tbody>
</table>
S22 vs. Hybrid Results (Contrast Ratio = 1:3, 1:9, 1:15, 1:27)

Maximum S22 Error: 0.62%
Maximum Hybrid Error: 0.42%

Maximum S22 Error: 0.69%
Maximum Hybrid Error: 0.15%

Maximum S22 Error: 0.70%
Maximum Hybrid Error: 0.16%

Maximum S22 Error: 0.71%
Maximum Hybrid Error: 0.18%
## S22 vs. Hybrid Error Comparison

<p>| Contrast Ratio | <strong>S22</strong> Maximum % Error ( \frac{\max |E_z(i, j, t) - E_{z,\text{ref}}(i, j, t)|}{\max |E_{z,\text{ref}}(i_{\text{source}} - i_{\text{offset}}, j_{\text{source}}, t)|} ) | <strong>Hybrid (S24)</strong> Maximum % Error ( \frac{\max |E_z(i, j, t) - E_{z,\text{ref}}(i, j, t)|}{\max |E_{z,\text{ref}}(i_{\text{source}} - i_{\text{offset}}, j_{\text{source}}, t)|} ) | Hybrid Improvement ( \frac{|\text{Error}<em>{S22} - \text{Error}</em>{S24}|}{\text{Error}_{S22}} ) |
|----------------|-------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|---------------|
| 1:3            | 0.6168 %                                                                                       | 0.4202 %                                                                                       | 32 %          |
| 1:9            | 0.6971 %                                                                                       | 0.1803 %                                                                                       | 74 %          |
| 1:15           | 0.7036 %                                                                                       | 0.1614 %                                                                                       | 77 %          |
| 1:27           | 0.7061 %                                                                                       | 0.1550 %                                                                                       | 78 %          |</p>
<table>
<thead>
<tr>
<th>Contrast Ratio</th>
<th>Reference</th>
<th>S22</th>
<th>Hybrid</th>
<th>Hybrid Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Time</td>
<td></td>
<td>Total Time</td>
<td>Total Time</td>
</tr>
<tr>
<td>1:3</td>
<td>0.331</td>
<td>0.050</td>
<td>0.054</td>
<td>83.69%</td>
</tr>
<tr>
<td>1:9</td>
<td>7.780</td>
<td>0.326</td>
<td>0.316</td>
<td>95.94%</td>
</tr>
<tr>
<td>1:15</td>
<td>34.658</td>
<td>1.371</td>
<td>1.373</td>
<td>96.04%</td>
</tr>
<tr>
<td>1:27</td>
<td>3052.957</td>
<td>491.586</td>
<td>495.241</td>
<td>83.78%</td>
</tr>
</tbody>
</table>

*All simulations were run using MATLAB R2018a software on a 64-bit Intel® Xeon® CPU E5-2680 0 at 2.70 GHz, 2.70 GHz (2 processors) with 256 GB of RAM.*
## Memory Usage Breakdown

<table>
<thead>
<tr>
<th>Contrast Ratio</th>
<th>Memory (GB)</th>
<th>Hybrid Improvement</th>
<th>Memory Saved</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reference</td>
<td>S22</td>
<td>Hybrid</td>
</tr>
<tr>
<td>1:3</td>
<td>2.371</td>
<td>2.154</td>
<td>2.156</td>
</tr>
<tr>
<td>1:9</td>
<td>4.294</td>
<td>2.230</td>
<td>2.232</td>
</tr>
<tr>
<td>1:15</td>
<td>17.055</td>
<td>2.327</td>
<td>2.327</td>
</tr>
<tr>
<td>1:27</td>
<td>21.192</td>
<td>2.581</td>
<td>2.543</td>
</tr>
</tbody>
</table>

*All simulations were run using MATLAB R2018a software on a 64-bit Intel® Xeon® CPU E5-2680 0 at 2.70 GHz, 2.70 GHz (2 processors) with 256 GB of RAM.*
2D Subgridding – Multiple Subgrid Regions
Hybrid FDTD Problem Space & Results

2D Domain: 319 x 388 Coarse Cells

Coarse Grid

Subgrid CR=3

Subgrid CR=9

Subgrid CR=3

Gaussian Source

PML—10 Coarse Cells

Cells along y

Cells along x

Normalized % Error

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Achievements:

- Acceptable levels of error in the S22 and Hybrid domains.
- Acceptable levels of error with higher contrast ratios, up to 27.
- Significant speedup in CPU time utilizing subgridding methods.
- Significant reduction in memory usage.
- Successful implementation of multiple subgrid regions in a 2D domain.

Future Work:

- Integrating subgridding in a 3D computational domain to begin testing on realistic scenarios such as filter and antenna array problems.
- Implementing a hybrid 4th and 2nd order FDTD calculation of the electric and magnetic fields to increase accuracy of the simulations in the coarse domain.
- Subgrid Regions with higher contrast ratios, 30, 90, etc.
Publications


Achievements: ACES 2019 Student Paper Competition 3rd Place Winner, 2019 ACES Conference, Miami, FL.
Questions?

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FDTD Subgridding References

• References: