Simulation of a Nonlinear Frequency Multiplier using the FDTD Technique

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Introduction

- Common RF devices, such as mixers and detectors, employ nonlinear components to function.
- Increasingly, nonlinear effects are employed to improve energy efficiency and thermal properties of modern amplifiers.
- Simulation is challenging with nonlinear devices – frequency-domain approaches break down.
- Goal: use nonlinear lumped-element devices integrated in FDTD grid to demonstrate nonlinear effects relevant to RF communications.
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Nonlinear behavior occurs in diodes, transistors, and ferrites.
Introduction - Nonlinearity

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- Example: resistor and diode
  - I/V characteristic for resistor is defined by Ohm’s law:

\[ I = \frac{V}{R} \]  

Linear Equation
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  - I/V characteristic for diode is defined by an exponential function:
    \[ I_D = I_S \left( e^{\frac{V_D}{\eta V_T}} - 1 \right) \]
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\[ \begin{array}{c|c|c}
\text{Voltage (V)} & -2 & -1.5 \\
\text{Current (A)} & -2 & -1.5 \\
\end{array} \]

Resistor
\[ 1 \Omega \]

Ideal Diode

Nonlinearity

Exponential Equation
Example: Excite a diode with a sinusoidal voltage source, measure current.

\[ V_D(t) = a \cos(\omega t) \]

Substitute \( I_D = I_S \left[ e^{\frac{V_D}{\eta V_T}} - 1 \right] \)

\[ I_D(t) = I_S \left[ e^{\frac{a \cos(\omega t)}{\eta V_T}} - 1 \right] \]
Example: Excite a diode with a sinusoidal voltage source, measure current.

\[ V_D(t) = a \cos(\omega t) \]

Substitute \( I_D = I_S \left[ e^{\frac{V_D}{\eta V_T}} - 1 \right] \)

\[ I_D(t) = I_S \left[ e^{\frac{a \cos(\omega t)}{\eta V_T}} - 1 \right] \]

Replace exponential by Maclaurin series:

\[ I_D(t) = I_S \sum_{n=1}^{\infty} \frac{(a \cos(\omega t))^n}{n!} \]

Take first three terms:

\[ I_D(t) \approx I_S \left( \frac{a \cos(\omega t)}{\eta V_T} + \frac{a^2 (\cos(2\omega t) + 1)}{4\eta^2 V_T^2} + \frac{a^3 (\cos(3\omega t) - 3 \cos(\omega t))}{24\eta^3 V_T^3} \right) \]
Diode With Series Resistor

- **FDTD Domain:** 40 x 38 x 43 (65360) cells
  - CPML Boundaries – 10 cells with 8 cell air-buffer
  - Cell size - \(dx = dy = dz = 0.05\) mm
- **Excitation:** 10 V\(_{pp}\), 5 GHz sinusoidal

\[R = 50\Omega\]
Diode With Series Resistor: Time-Domain Results

\[ R = 50 \Omega \]

\[ R_S = 50 \Omega \]

\[ I \]

\[ V \]

The graph shows the source voltage and the voltage across the resistor for FDTD and ADS simulations. The plots are labeled as:
- Source Voltage
- Voltage Across Resistor (FDTD)
- Voltage Across Resistor (ADS)
Diode With Series Resistor: Frequency Domain Results

\[ R = 50 \Omega \]

\[ R_S = 50 \Omega \]

- Voltage Source
- Voltage Across Resistor (FDTD)
- Voltage Across Resistor (ADS)
Diode with Low Pass Filter

Voltage Source (Port 1)

\[ f = 5 \text{ GHz} \]
\[ V_s = 3 \text{ Vpp} \]
\[ R_s = 50\Omega \]

Low-pass Filter

\[ C_1 = 1.044 \text{ pF} \]
\[ L_1 = 1.882 \text{ nH} \]
\[ C_2 = 1.555 \text{ pF} \]

Diode

Load (Port 2)

\[ R = 50\Omega \]
Diode with Low Pass Filter

Voltage Source (Port 1)  Low-pass Filter  Diode  Load (Port 2)

\( f = 5 \text{ GHz} \)
\( V_s = 3 \text{ Vpp} \)
\( R_s = 50 \Omega \)

\( C_1 = 1.044 \text{ pF} \)
\( L_1 = 1.882 \text{ nH} \)
\( C_2 = 1.555 \text{ pF} \)

\( R = 50 \Omega \)
Conclusions and Future Work

- FDTD provides useful simulations of nonlinear components integrated in microwave circuits.
- Future work: analyze the results using the nonlinear X-parameters.
References


